LAST Name: Answer:

First Name:

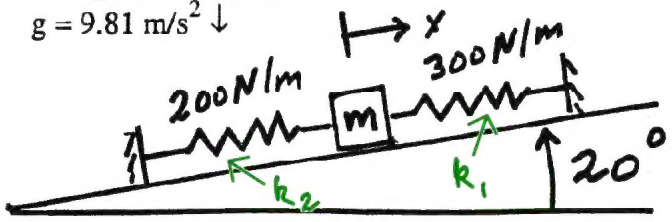
Student Number:

MIE 200F - Quiz number 8a - November 28/00
quiz duration = 25 minutes

The 10-kg mass is sliding on a hill with zero friction, as shown. The origin is located a the point where the sum of all forces on the mass equals zero. Assume that both springs are in tension at all times.

(a) What is the natural frequency of vibration of the system, expressed in Hertz?

(b) If the peak-to-peak amplitude of the vibrations is 6 meters, what is the maximum speed that is achieved by the mass during each cycle of vibration?

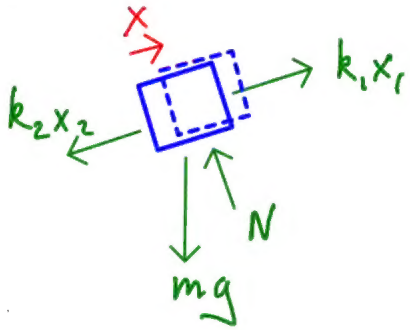


The equation $\ddot{x} + \omega_n^2 x = 0$, has the solution:

$$x(t) = C_1 \cos(\omega_n t) + C_2 \sin(\omega_n t)$$
$$= C_3 \sin(\omega_n t + \varphi)$$

$\tau = 1/f$ $\quad f = \omega_n / 2\pi$

(a) Springs are in parallel:



$$\Sigma F_x = m\ddot{x} = -300x - 200x$$

$$\ddot{x} + \frac{500}{10\,kg} x = 0$$

$$\ddot{x} + 50x = 0$$

$$\ddot{x} + \omega_n^2 x = 0$$

$$x = C_3 \sin(\omega_n t + \varphi)$$

$$\omega_n^2 = 50 \Rightarrow \omega_n = \sqrt{50}$$

$$f = \sqrt{50}/2\pi = \boxed{1.13\ Hz}$$

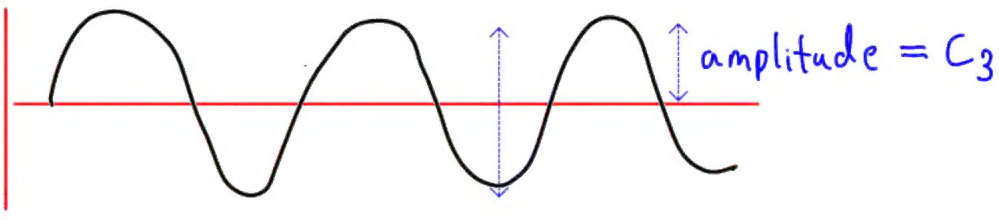
(b) peak-to-peak amplitude = 6 m.

$$\Rightarrow C_3 = 6/2 = 3$$

$$x = 3 \sin(\sqrt{50}\, t + \varphi)$$

$$\dot{x} = 3\sqrt{50} \cos(\sqrt{50}\, t + \varphi)$$

$$\Rightarrow \dot{x}_{max} = 3\sqrt{50} = \boxed{21.2\ m/s}$$



LAST Name: Answer.

First Name: _____

Student Number: _____

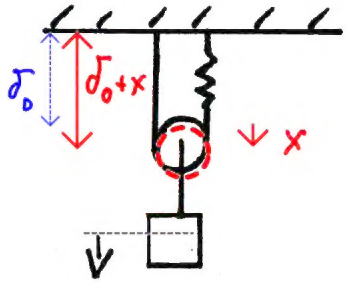
MIE 200F - Quiz number 8b - November 29/00
quiz duration = 25 minutes

The mass of 5 kg is supported from the ceiling by a rope, spring, and wheel as shown. Assume that the rope, spring and wheel have zero mass. The spring constant is 10 N/m.

(a) What will be the equilibrium extension of the spring when the mass is hanging from the system, with zero acceleration?

(b) What will be the period of the natural vibration of the system?

$g = 9.81\ m/s^2 \downarrow$



$I_{c\ of\ g}$ for a rod $= m L^2/12$ $\qquad I_o = I_{c\ of\ g} + m d^2$

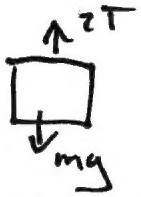
$\Sigma F = m a_{c\ of\ g}$ $\qquad \Sigma M_o = I_o \alpha$

The equation $\ddot{x} + \omega_n^2 x = 0$, has the solution:

$x(t) = C_1 \cos(\omega_n t) + C_2 \sin(\omega_n t)$

$\tau = 1/f$ $\qquad f = \omega_n / 2\pi$

(a) At equib,



$2T = mg$

$(2)(10\ N/m)(x_0) = (5)(9.81) \Rightarrow x_0 = 2.45\ m.$

(b) Let x = displacement from equib.

⇒ mass goes down a distance x

⇒ spring is stretched by an extra 2x

⇒ tension is increased by 2kx

⇒ upward (restoring) force $= (2)(2kx) = 4kx$

$\Sigma F = m\ddot{x} = -4kx$

$\ddot{x} + 4k/m = 0$

$\ddot{x} + \omega_n^2 = 0$

$\omega_n = \sqrt{4k/m} = \sqrt{(4)(10)/5} = \boxed{\sqrt{8}\ \ s^{-1}}$

$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{8}} = \boxed{2.22\ s}$

This particular pulley arrangement has increased the spring "effectiveness" by a factor of 4.

LAST Name: _____

First Name: _____

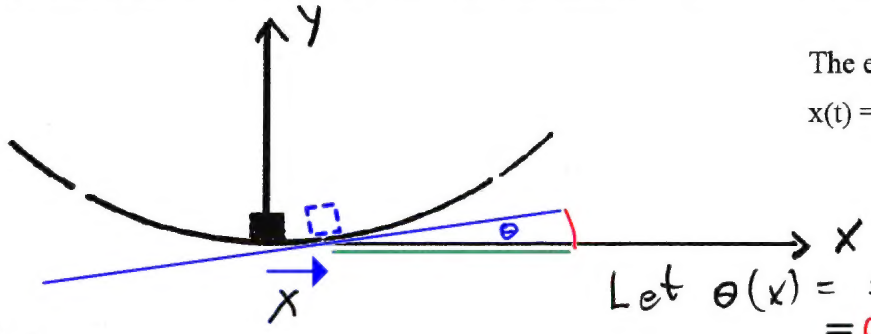
Student Number: _____

MIE 200F - Quiz number 8b - November 30/99
quiz duration = 25 minutes

The bottom of a valley between two hills is shaped like a parabola, such that $y = 0.06\,x^2$ (SI units). A very small, dense rock in the shape of a cube lies in the bottom of the valley, in equilibrium at $\{x,y\} = \{0,0\}$. The rock has a mass of 5 kg. Assume that friction is negligible. Acceleration due to gravity is $g = 9.81$ m/s^2 ↓.

(a) Assume that the rock is displaced a small positive distance x to the right from its equilibrium position. Draw a free body diagram showing the magnitude and direction of forces that will then act on the mass. Evaluate any angles in terms of the displacement x. What will be the resulting force that tries to push the block back to the left?

(b) What is the natural frequency (Hertz) at which this mass would oscillate back and forth at the bottom of the valley?



The equation $\ddot{x} + \omega_n^2 x = 0$, has the solution:

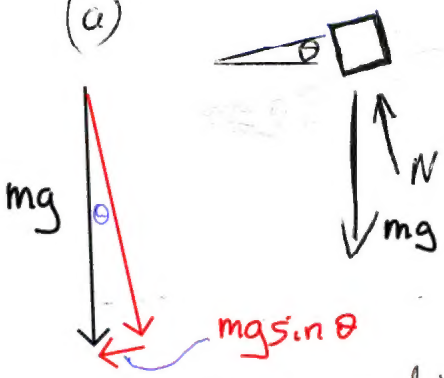
$x(t) = C_1 \cos(\omega_n t) + C_2 \sin(\omega_n t)$

$\tau = 1/f \qquad f = \omega_n / 2\pi$

Let $\theta(x)$ = slope at position x (in radians)
= arclength/radius = dy/dx

θ = slope = $\frac{dy}{dx} = 0.12x$

(a)



Displace mass a distance x

Force pushing back to left = $-(mg)(\sin\theta)$

$= -(5)(9.81)(0.12x)$

$F = -5.886\,x = m\ddot{x}$

(b) $5\ddot{x} + 5.886x = 0$

$\omega_n = \sqrt{5.886/5} = 1.085\ s^{-1}$

$f = \omega_n / 2\pi = 0.173$ Hz